Реш-е соотн-ний Гюгонио

$\frac{u_0}{\tau_0} = \frac{u_1}{\tau_1}$ (u - поток массы); $p = (\gamma-1)\tau\varepsilon$ - для ид. газа

$\frac{u_0^2}{\tau_0} + p_0 = \frac{u_1^2}{\tau_1} + p_1$; $\frac{u_0}{\tau_0}\left(\varepsilon_0^2 + \frac{u_0^2}{2} p_0\tau_0\right) = \frac{u_1}{\tau_1}\left(\varepsilon_1^2 + \frac{u_0^2}{2} + p_1\tau_1\right)$

1) Уд. волна $\Rightarrow m = \frac{u_1}{\tau_1} = \frac{u_0}{\tau_0} \neq 0 \Rightarrow \varepsilon_1 + \frac{u_1^2}{2} + p_1\tau_1 = \varepsilon_0 + \frac{u_0^2}{2} + p_0\tau_0$,

где $\varepsilon_1 = \frac{p_1\tau_1}{\gamma-1}$; $\varepsilon_0 = \frac{p_0\tau_0}{\gamma-1} \Rightarrow p_1\tau_1\left(\frac{1}{\gamma-1}+1\right) + \frac{u_1^2}{2} = p_0\tau_0\left(\frac{1}{\gamma-1}+1\right) + \frac{u_0^2}{2} \Rightarrow$

$\Rightarrow \frac{\gamma}{\gamma-1} p_1\tau_1 + \frac{u_1^2}{2} = \frac{\gamma}{\gamma-1} p_0\tau_0 + \frac{u_0^2}{2} \Big| :\tau_1 \Rightarrow \frac{\gamma}{\gamma-1} p_1 + \frac{u_1^2}{2\tau_1} = \frac{\gamma}{\gamma-1}\frac{p_0\tau_0}{\tau_1} + \frac{u_0^2}{2\tau_1} \Rightarrow$

$\Rightarrow \frac{\gamma}{\gamma-1} p_1 = \frac{1}{\tau_1}\left(\frac{\gamma}{\gamma-1} p_0\tau_0 + \frac{u_0^2}{2} - \frac{u_1^2}{2}\right)$ (*); $(2)\cdot\frac{\gamma}{\gamma-1} - (*) \Rightarrow$

$\Rightarrow 0 = \frac{1}{u_1}\left(\frac{\gamma}{\gamma-1} p_0\tau_0 + \frac{u_0^2}{2} - \frac{u_1^2}{2}\right) - \frac{\gamma}{\gamma-1}\left(\frac{u_0^2}{\tau_0} + p_0 - \frac{u_1^2}{\tau_1}\right)$; $\frac{\gamma}{\gamma-1} p_1 = \frac{\gamma}{\gamma-1}\left(\frac{u_0^2}{\tau_0} + p_0 - \frac{u_1^2}{\tau_1}\right)$

Из (1): $\frac{1}{\tau_1} = \frac{u_0}{\tau_0}\cdot\frac{1}{u_1}$; $\Rightarrow \frac{u_0}{\tau_0}\frac{1}{u_1}\left(\frac{\gamma}{\gamma-1} p_0\tau_0 + \frac{u_0^2}{2} - \frac{u_1^2}{2}\right) - \frac{\gamma}{\gamma-1}\left(\frac{u_0^2}{\tau_0} + p_0 - \frac{u_1^2 u_0}{\tau_0 u_1}\right) = 0$

$\frac{u_0}{\tau_0}\left(\frac{\gamma}{\gamma-1} p_0\tau_0 + \frac{u_0^2}{2} - \frac{u_1^2}{2}\right) - \frac{\gamma}{\gamma-1}\left(\left(\frac{u_0^2}{\tau_0} + p_0\right)u_1 - \frac{u_1^2 u_0}{\tau_0}\right) = 0$ .....

$\Rightarrow u_1 = \frac{\gamma-1}{\gamma+1} u_0 + \frac{2\gamma}{\gamma+1}\cdot\frac{p_0\tau_0}{u_0}$ - уд. волна

$\tau_1 = \frac{u_1}{u_0}\tau_0$; $p_1 = \frac{u_0^2}{\tau_0} + p_0 - \frac{u_1^2}{\tau_1}$

Ударная адиабата (адиабата Гюгонио в пл-ти (V,p)

$\frac{u_1}{\tau_1} = \frac{u_0}{\tau_0} = m \neq 0$ (уд. волна); $\frac{u_1^2}{\tau_1} + p_1 = \frac{u_0^2}{\tau_0} + p_0 \Rightarrow u_1 m + p_1 = u_0 m + p_0$
$m(u_1 - u_0) = p_0 - p_1$

$\Rightarrow m = \frac{p_0 - p_1}{u_1 - u_0}$; $u_1 - u_0 = (D - V_1) - (D - V_0) = V_0 - V_1 \Rightarrow m = \frac{p_0 - p_1}{V_0 - V_1}$

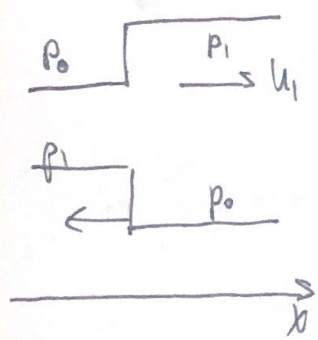
$$\frac{\tau_1}{\tau_0} = \frac{(\gamma+1)p_0 + (\gamma-1)p_1}{(\gamma-1)p_0 + (\gamma+1)p_0} = \frac{p_0 + \frac{\gamma-1}{\gamma+1}p_1}{p_1 + \frac{\gamma-1}{\gamma+1}p_0} = \left\{\mu^2 = \frac{\gamma-1}{\gamma+1}\right\} = \frac{p_0 + \mu^2 p_1}{p_1 + \mu^2 p_0}$$

$$\frac{u_1^2}{\tau_1} + p_1 = \frac{u_0^2}{\tau_0} + p_0 \Rightarrow \left(\frac{u_1}{\tau_1}\right)^2 \tau_1 + p_1 = \left(\frac{u_0}{\tau_0}\right)^2 \tau_0 + p_0 \Rightarrow m^2 \tau_1 + p_1 = m^2 \tau_0 + p_0 \Rightarrow$$

$$\Rightarrow m^2 = \frac{p_0 - p_1}{\tau_1 - \tau_0} = \frac{p_0 - p_1}{\tau_0\left(\frac{\tau_1}{\tau_0} - 1\right)} = \frac{p_0 - p_1}{\tau_0\left(\frac{p_0 + \mu^2 p_1}{p_1 + \mu^2 p_0} - 1\right)} = \frac{(p_0 - p_1)(p_1 + \mu^2 p_0)}{\tau_0(p_0 + \mu^2 p_1 - p_1 - \mu^2 p_0)} =$$

$$= \frac{\cancel{(p_0 - p_1)}(p_1 + \mu^2 p_0)}{\tau_0 \cancel{(p_0 - p_1)} (1 - \mu^2)} = \frac{p_1 + \mu^2 p_0}{\tau_0(1-\mu^2)} \Rightarrow m = \pm\sqrt{\frac{p_1 + \mu^2 p_0}{\tau_0(1-\mu^2)}}$$

Для уд. волн всегда $p_1 > p_0$



уд. волна движ-ся вправо(+)

уд. волна движ-ся влево(−)

$m = u_1\rho_1 = u_0\rho_0$, $m > 0$ "+" $\Rightarrow u_1\rho_1 > 0 \Rightarrow$

$\Rightarrow u_1 > 0$, $u_1 = D - v_1$

$m < 0$ "−" $\Rightarrow u_1\rho_1 < 0 \Rightarrow u_1 < 0$

$m = \frac{p_0 - p_0}{v_0 - v_1}$, $v_0 - v_1 = \frac{1}{m}(p_0 - p_1)$

$m > 0$, $p_0 - p_1 < 0 \Rightarrow v_0 < v_1$ (m"+")

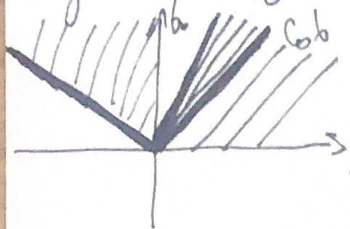
$m < 0$, $p_0 - p_1 < 0 \Rightarrow v_0 > v_1$ (m"−")

$$v_1 = \frac{1}{\pm\sqrt{\ldots}}(p - p_0) + v_0; \quad \frac{dv}{dp} = \frac{1}{\pm\sqrt{\frac{p+\mu^2 p_0}{\tau_0(1-\mu^2)}}} \pm (p - p_0)\left(-\frac{1}{2}\right)\left(\frac{p + \mu^2 p_0}{\tau_0(1-\mu^2)}\right)^{-\frac{3}{2}} \frac{1}{\tau_0(1-\mu^2)}$$

$$\left.\frac{dv}{dp}\right|_{p_0} = \pm\sqrt{\frac{\tau_0(1-\mu^2)}{p_0(1+\mu^2)}} = \pm\sqrt{\frac{1}{\frac{\gamma p_0}{\rho_0}\rho_0^2}} = \pm\frac{1}{c_0\rho_0}; \quad c_0^2 = \gamma\frac{p_0}{\rho_0} = \gamma p_0 \tau_0$$

$$\frac{1 - \frac{\gamma-1}{\gamma+1}}{1 + \frac{\gamma-1}{\gamma+1}} = \frac{1}{\gamma} = \frac{1-\mu^2}{1+\mu^2}$$

Адиабата Пуассона в пл-ти (v,p)



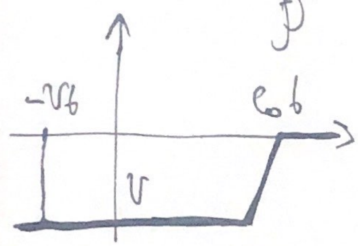
$J^- = v - \frac{2c}{\gamma-1} = const$; $C^+: \frac{dx}{dt} = v + c$

$v = v_0 + \frac{2}{\gamma-1}(c - c_0)$

$c = c_0 + \frac{\gamma-1}{2}(v - v_0)$

$C^+: \frac{dx}{dt} = \frac{\gamma+1}{2} v - \frac{\gamma-1}{2} v_0 + c_0$

J^+



$v = v_0 + \frac{2c_0}{\gamma-1}\left(\frac{c}{c_0} - 1\right)$

$c = \sqrt{\gamma p h}$

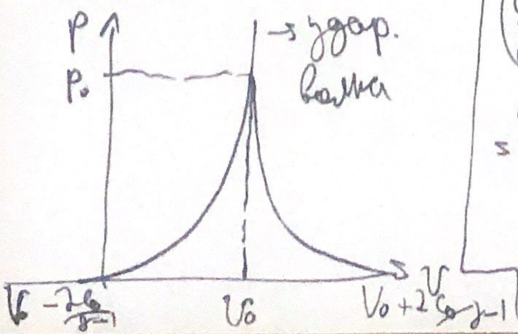
$$v = v_0 + \frac{2c_0}{\gamma-1}\left(\sqrt{\frac{\gamma p h}{\gamma p_0 h_0}} - 1\right) = v_0 + \frac{2c_0}{\gamma-1}(\dots) \; \text{(S)}$$

$\frac{p}{\rho^\gamma} = \frac{p_0}{\rho_0^\gamma} = const$; $p h^\gamma = p_0 h_0^\gamma$; $\frac{h}{h_0} = \left(\frac{p_0}{p}\right)^{1/\gamma}$

$$\text{(S)} \; v_0 + \frac{2c_0}{\gamma-1}\left(\left(\frac{p}{p_0}\left(\frac{p_0}{p}\right)^{1/\gamma}\right)^{\frac{1}{2}} - 1\right) = v_0 + \frac{2c_0}{\gamma-1}\left(\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{2\gamma}} - 1\right)$$

$\vec{J^+}$: $v = v_0 + \frac{2c}{\gamma-1}\left(\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{2\gamma}} - 1\right)$; $\left(\frac{\gamma-1}{2c_0}(v - v_0) + 1\right)^{\frac{2\gamma}{\gamma-1}} = \frac{p}{p_0}$;

$$\left(\frac{\gamma-1}{2c_0}(v - v_0) + 1\right)^{\frac{2\gamma}{\gamma-1}} p_0 = p$$



$p < p_0$ волна разрежения

$$\left(\frac{dv}{dp}\right)\Big|_{p=p_0} = \left(\frac{2c_0}{\gamma-1}\cdot\frac{\gamma-1}{2\gamma}(p)^{\frac{\gamma-1}{2\gamma}-1}\cdot\frac{1}{p_0^{\frac{\gamma-1}{2\gamma}}}\right)\Big|_{p=p_0} =$$

$$= \frac{2c_0}{\gamma-1}\cdot\frac{\gamma-1}{2\gamma}\cdot\frac{1}{p_0} = \frac{c_0}{\gamma p_0} = \frac{c_0}{\gamma p_0} = \sqrt{\frac{\gamma p_0}{\rho_0}}\cdot\frac{1}{\gamma p_0} =$$

$$= \frac{1}{\sqrt{\gamma p_0 \rho_0}}\cdot\frac{\rho_0}{\rho_0} = [illegible] = \frac{1}{c_0 \rho_0}$$ т.к. $\frac{\rho_0}{\gamma p_0} = \frac{1}{c_0}$